Frequency Localization for Weil Operator Certificates

Scott Schubr
Luminary Oddities

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Abstract

We give a frequency-block enclosure for a constrained theta-weighted Weil operator. The retained paired-prime operator is translation invariant before the theta multipliers are applied. Splitting at that intermediate stage eliminates the cross-frequency blocks exactly. A finite-rank approximation of the remaining low-frequency map then has a complete operator error whose high-frequency term is proportional to the reciprocal digamma symbol, rather than its inverse square root. We retain the signed finite interaction matrix and enclose its resolvent Gram by full residuals. We also prove that the inverse-square-root bound for the raw resolvent output is asymptotically sharp; an abstract conditional extension covers valid null-space deflations. These results specify improved sufficient localization budgets and a limitation of raw-output approximation.

1 Introduction

A finite matrix can certify a lower bound for an infinite-dimensional operator only when its omitted and mixed terms are paid. In a weighted Weil realization, the positive reference controls a logarithmic Fourier moment, while the arithmetic perturbation is a sum of paired translations between theta multipliers. Approximating the entire resolvent output first loses the particular algebra of those translations. We instead split the retained arithmetic form in frequency before spatial localization.

The main result is an explicit finite-rank enclosure in operator norm, with a signed one-sided refinement. The finite matrix retains its negative part. Its resolvent Gram can be bounded using the entire residual covariance, without assuming that a Galerkin residual equals a full residual. A second result proves the sharp leading asymptotic of the raw high-frequency norm. Thus the improved certificate is a consequence of cancellation at the operator level, rather than an improved raw-output estimate.

The constrained realization, its chosen core closure and paired arithmetic tail were developed in the author's working notes [3]. The signed finite interaction matrix and full residual-Gram enclosure already occur in [4]; they are reproduced here to make the certificate self-contained. The present contribution is to split the arithmetic sandwich before localization, orient the finite approximation as a frequency projection after a spatial cutoff, and obtain a reciprocal-symbol error together with a sharp raw-tail asymptotic.

The ingredients are classical: Fourier multipliers and orthogonal projections, real-phase Taylor remainders, Hilbert–Schmidt bounds, resolvent identities and positive-form representation; see [2, 1]. For other operator realizations of Weil's form, see Suzuki [5]; full-arithmetic constructions also appear in [6]. Those constructions are context, rather than proof dependencies for the estimates below. We define the weighted realization and prove the bounds needed here directly. The resulting contribution is a quantitative localization theorem for this specified operator.

2 The weighted realization and its explicit bounds

All Hilbert spaces are complex, with inner products conjugate-linear in the first argument. Fourier transformation is

$$\widehat{f}(\xi) = \int_{\mathbb{R}} e^{-iu\xi} f(u)\,\mathrm{d}u, \qquad \|f\|^2 = \int_{\mathbb{R}} |\widehat{f}(\xi)|^2 \frac{\mathrm{d}\xi}{2\pi}.$$

Let $\vartheta(x) = \sum_{n\in\mathbb{Z}} e^{-\pi n^2 x}$. Define the Riemann theta kernel, in the present spatial normalization, by

$$\psi(u) = 2\Phi_{\mathrm{RT}}(u/2), \qquad \Phi_{\mathrm{RT}}(t) = \sum_{n=1}^{\infty} (2\pi^2 n^4 e^{9t} - 3\pi n^2 e^{5t}) e^{-\pi n^2 e^{4t}}. \tag{1}$$

The series and all derivatives converge locally uniformly for real arguments. It is even after the indicated scaling. To check this normalization, put $b(u) = e^{u/2}(\vartheta(e^{2u}) - 1)$. Poisson summation gives

$$b(u) - b(-u) = e^{-u/2} - e^{u/2}.$$

Direct differentiation gives $\psi = \frac{1}{2}(\partial_u^2 - 1/4)b$, proving evenness. For $u \geq 0$ the individual terms of ψ are positive, since $\pi n^2 e^{2u} > 3/2$. Hence ψ is smooth and strictly positive everywhere.

Put

$$c(u) = \cosh(u/2), \quad w = \psi c, \quad s = \sqrt{\psi/c}, \quad \mathcal{K} = \{h \in L^2(\mathbb{R}) : \langle \sqrt{w}, h\rangle = \langle \sqrt{w}\tanh(u/2), h\rangle = 0\}.$$

The two row vectors are in L^2 and linearly independent. Under the weighted unitary transformation $h = \sqrt{w}\,g$, the auxiliary physical test is $\psi g = sh$. We retain exactly these two rows.

Write $\psi_0(z) = \Gamma'(z)/\Gamma(z)$ for the digamma function, to distinguish it from the theta kernel. Define

$$m_0 = \psi_0(1/4) - \log\pi, \qquad a(\xi) = \Re\psi_0(1/4 + i\xi/2) - \psi_0(1/4). \tag{2}$$

The standard digamma series and asymptotic [1, §§5.7,5.11] give, with $t = |\xi|/2$,

$$a(\xi) = \sum_{j=0}^{\infty} \frac{t^2}{(j+1/4)((j+1/4)^2 + t^2)}, \qquad a(\xi) = \log|\xi| + O(1) \quad (|\xi| \to \infty). \tag{3}$$

In particular a is nonnegative, even, and strictly increasing in $|\xi| > 0$.

Let $\mathcal{C} = C_c^\infty(\mathbb{R}) \cap \mathcal{K}$. It is dense in $\mathcal{K}$: approximate in L^2 by compact smooth functions and repair their two row errors using two fixed compact smooth bumps whose row matrix is invertible. Such bumps exist by linear independence of the smooth row functions. Define on $\mathcal{C}$

$$\mathfrak{a}[h] = \int_{\mathbb{R}} a(\xi)|\widehat{sh}(\xi)|^2 \frac{\mathrm{d}\xi}{2\pi}.$$

This form is closable. Indeed the maximal graph map $h \mapsto a(D)^{1/2}(sh)$ is closed, since multiplication by s is bounded and the Fourier multiplier is closed. Restriction of its nonnegative closed form is closable. Let $\mathcal{V}$ be the closure of $\mathcal{C}$ in its form norm, and let $A \geq 0$ be its associated selfadjoint operator. We use this chosen core closure throughout, without asserting equality with a maximal form domain.

Let $S_b f(u) = f(u+b)$ and $c_n = \Lambda(n)/\sqrt{n}$, where Λ is the von Mangoldt function. On $\mathcal{K}$ define

$$B = P_{\mathcal{K}} \left[m_0 M_s^2 - \sum_{n=2}^{\infty} c_n M_s (S_{\log n} + S_{-\log n}) M_s \right] \Big|_{\mathcal{K}}, \qquad H = A + B. \tag{4}$$

The bounds below prove convergence in operator norm. This defines the theta-weighted constrained realization directly. All results concern this specified H and its bounded perturbations.

Lemma 2.1 (Explicit theta and arithmetic budgets)**.** *For $u \geq 0$,*

$$\psi(u) \leq 8\pi^2 e^{9u/2-\pi e^{2u}}, \qquad s(u)^2 \leq 16\pi^2 e^{4u-\pi e^{2u}}.$$

Consequently $\|s\|_\infty \leq 3$ and, for $R > 0$,

$$\sup_{|u|>R} s(u) \leq \sigma_R := \min\{3, 13e^{2R-(3/2)e^{2R}}\}.$$

If B_M retains the same m_0 term and $2 \leq n \leq M$, then, for integer $M \geq 1$,

$$\|B - B_M\| \leq \delta_M := \frac{640(M+1)^2}{20^{M+1}}. \tag{5}$$

Moreover $|m_0| < 6$ and $a(L) \geq \frac{1}{2}\log(2L)$ for $L \geq 1/2$.

Proof. Put $r = \pi e^{2u} \geq \pi$. Discarding the negative coefficient in (1) gives

$$\psi(u) \leq 4\pi^2 e^{9u/2-r} \sum_{n\geq 1} n^4 e^{-r(n^2-1)}.$$

For $n \geq 2$, $n^2 - 1 \geq 3(n-1)$. With $q = e^{-3\pi} < 1/100$, the sum is at most

$$\sum_{n\geq 1} n^4 q^{n-1} = \frac{1 + 11q + 11q^2 + q^3}{(1-q)^5} < 2.$$

This proves the first bound. Divide by $c(u) \geq e^{u/2}/2$ for the second. The function $x^2 e^{-\pi x}$ decreases for $x \geq 1$; using $\pi^2 < 10$ and $e^\pi > 20$ gives $s^2 < 8 < 9$. Its square root and $4\pi < 13$, $\pi > 3$ give σ_R; reflection covers $u < 0$.

For $b \geq 0$, a weighted translation has norm $\sup_u s(u)s(u+b)$. Write $v = |u| + |u+b| \geq b$. The two theta bounds and arithmetic–geometric mean give

$$s(u)s(u+b) \leq 16\pi^2 e^{2v-\pi e^v} \leq 16\pi^2 e^{2b-\pi e^b},$$

since the latter exponent decreases for $v \geq 0$. Thus the channel with $b = \log n$ has norm at most $16\pi^2 n^2 e^{-\pi n}$. Since $c_n \leq 1$, the paired tail is at most $32\pi^2 \sum_{n>M} n^2 e^{-\pi n}$. Put $k = M + 1 \geq 2$ and use $e^{-\pi} < 1/20$:

$$\sum_{n\geq k} n^2 e^{-\pi n} \leq k^2 20^{-k} \sum_{j\geq 0} (j+1)^2 20^{-j} < 2k^2 20^{-k}.$$

Together with $\pi^2 < 10$, this proves (5), including norm convergence and bounded selfadjointness of B. The bound $c_n \leq 1$ follows from $\Lambda(n) \leq \log n \leq \sqrt{n}$. The classical value $\psi_0(1/4) = -\gamma - \pi/2 - 3\log 2$ gives $|m_0| < 6$. Finally, for $L \geq 1/2$ put $t = L/2$, $J = \lfloor t - 1/4 \rfloor \geq 0$. Retaining $0 \leq j \leq J$ in (3) gives

$$a(L) \geq \frac{1}{2} \sum_{j=0}^{J} \frac{1}{j + 1/4} \geq \frac{1}{2}\log(4J+5) \geq \frac{1}{2}\log(2L).$$

At endpoints the same weak inequalities suffice. This completes all budgets. □

3 The frequency-block enclosure

Initially let $\mathcal{S} = \mathcal{K}$, $A_{\mathcal{S}} = A$, and $B_{\mathcal{S}} = B$. More generally the following proof works on any closed receiving subspace with a nonnegative reference satisfying the same form identity and the compressed factorization (4); valid null-space compression is addressed in Section 5. Fix $\varepsilon > 0$ and put

$$R_\varepsilon = (A_{\mathcal{S}} + \varepsilon)^{-1/2}, \quad J = M_s R_\varepsilon, \quad j_\varepsilon = 3/\sqrt{\varepsilon}, \quad K_\varepsilon = R_\varepsilon(-B_{\mathcal{S}})R_\varepsilon.$$

For unit $h \in \mathcal{S}$,

$$\|Jh\| \le j_\varepsilon, \qquad \int a(\xi)|\widehat{Jh}(\xi)|^2 \frac{\mathrm{d}\xi}{2\pi} = \mathfrak{a}[R_\varepsilon h] \le 1. \tag{6}$$

The form inequality $H|_{\mathcal{S}} \ge -\varepsilon I$ is equivalent to $K_\varepsilon \le I$, by the invertible form-domain parametrization through R_ε.

Let Q_L be Fourier projection to $[-L, L]$. For fixed M set

$$V_M = -m_0 I + \sum_{2 \le n \le M} c_n (S_{\log n} + S_{-\log n}), \quad v_M(\xi) = -m_0 + 2 \sum_{2 \le n \le M} c_n \cos(\xi \log n),$$

$$d_M = |m_0| + 2 \sum_{2 \le n \le M} c_n \le 2M + 4, \qquad b_M(L) = \max\left\{0, \sup_{|\xi| > L} \frac{v_M(\xi)}{a(\xi)}\right\}.$$

Thus $\|V_M\| \le d_M$ and $b_M(L) \le d_M / a(L)$.

For $R, L > 0$ and integer $d \ge 0$, define the finite-rank map E_d by

$$\widehat{E_d f}(\xi) = 1_{[-L,L]}(\xi) \sum_{k=0}^{d} \frac{(-i\xi)^k}{k!} \int_{-R}^{R} u^k f(u)\, \mathrm{d}u, \qquad F_d = E_d J. \tag{7}$$

Write

$$b_d = \sqrt{\frac{2}{\pi}} \frac{(RL)^{d+3/2}}{(2d+3)(d+1)!}, \quad \tau_0 = \frac{\sigma_R}{\sqrt{\varepsilon}} + b_d j_\varepsilon, \quad e_0 = d_M \tau_0 (2 j_\varepsilon + \tau_0). \tag{8}$$

Theorem 3.1 (Complete frequency-block error)**.** *With the preceding definitions,*

$$\|K_\varepsilon - F_d^* V_M F_d\| \le \frac{\delta_M}{\varepsilon} + \frac{d_M}{a(L)} + e_0, \tag{9}$$

$$K_\varepsilon \le F_d^* V_M F_d + \left(\frac{\delta_M}{\varepsilon} + b_M(L) + e_0\right) I. \tag{10}$$

Every bound acts on the entire receiving Hilbert space, including its infinite complement.

Proof. The retained arithmetic sandwich is $K_{\varepsilon,M} = J^* V_M J$, and (5) gives $\|K_\varepsilon - K_{\varepsilon,M}\| \le \delta_M/\varepsilon$. The multiplier v_M commutes with $Q = Q_L$. Therefore

$$J^* V_M J = (QJ)^* V_M (QJ) + ((I-Q)J)^* V_M ((I-Q)J). \tag{11}$$

The cross-frequency terms are exactly zero. From (6), the high summand has norm at most $d_M/a(L)$, while its quadratic form is at most $b_M(L)\|h\|^2$, directly by integrating $v_M|\widehat{Jh}|^2$ on $|\xi| > L$.

Let P_R be spatial projection to $[-R, R]$. The real-phase integral Taylor remainder gives

$$\left| e^{-iu\xi} - \sum_{k=0}^{d} \frac{(-iu\xi)^k}{k!} \right| \le \frac{|u\xi|^{d+1}}{(d+1)!}.$$

Integrating its square on $[-R,R]\times[-L,L]$ with measure $\mathrm{d}u\,\mathrm{d}\xi/(2\pi)$ proves $\|Q_LP_R-E_d\|\le b_d$. In particular no exponential phase factor is needed. The ordering of the two cutoffs matters:

$$Q_LJ-F_d=Q_L(I-P_R)J+(Q_LP_R-E_d)J.$$

The theta tail bounds the first term by $\sigma_R/\sqrt{\varepsilon}$, hence $\|Q_LJ-F_d\|\le\tau_0$ and $\|F_d\|\le j_\varepsilon+\tau_0$. Expanding the difference of the two low-band products bounds it by $d_M\tau_0(2j_\varepsilon+\tau_0)$. Combining with the arithmetic tail and (11) proves both inequalities. Spatial localization is applied to J, not to an assumed spatially localized Q_LJ. □

4 The signed finite certificate and full residuals

Define $Y:\mathbb{C}^{d+1}\to L^2$ and $Q_0:\mathbb{C}^{d+1}\to\mathcal{S}$ by

$$\widehat{Ye_k}(\xi)=1_{[-L,L]}(\xi)(-i\xi)^k/k!,\qquad Q_0e_k=P_{\mathcal{S}}[s(u)1_{[-R,R]}(u)u^k].$$

All rows and columns are in their declared Hilbert spaces, and $F_d=YQ_0^*R_\varepsilon$. Put

$$C=Y^*V_MY,\qquad G=Q_0^*(A_{\mathcal{S}}+\varepsilon)^{-1}Q_0. \tag{12}$$

Then

$$C_{jk}=\int_{-L}^{L}v_M(\xi)\frac{(i\xi)^j(-i\xi)^k}{j!k!}\frac{\mathrm{d}\xi}{2\pi},\qquad F_d^*V_MF_d=R_\varepsilon Q_0CQ_0^*R_\varepsilon.$$

These finite polynomial–cosine integrals retain all off-diagonal entries and the sign of C.

Lemma 4.1 (Residual Gram enclosure)**.** *For any $X:\mathbb{C}^{d+1}\to\mathrm{Dom}(A_{\mathcal{S}})$, let*

$$E=Q_0-(A_{\mathcal{S}}+\varepsilon)X,\quad D=Q_0^*X+X^*Q_0-X^*(A_{\mathcal{S}}+\varepsilon)X.$$

Then

$$G-D=E^*(A_{\mathcal{S}}+\varepsilon)^{-1}E,\qquad 0\le G-D\le E^*E/\varepsilon.$$

Proof. Expand the right-hand product. Its four terms are respectively G, $-Q_0^*X$, $-X^*Q_0$ and $X^*(A_{\mathcal{S}}+\varepsilon)X$. The spectral bound $0\le(A_{\mathcal{S}}+\varepsilon)^{-1}\le\varepsilon^{-1}I$ proves the enclosure. No Galerkin orthogonality is assumed. □

Corollary 4.2 (A complete sufficient lower certificate)**.** *Suppose Hermitian matrices $\overline{C}\ge C$ and $\overline{G}\ge D+E^*E/\varepsilon$ include all enclosure errors, with $\overline{G}>0$. If $\beta\ge0$ and*

$$\overline{C}\le\beta\overline{G}^{-1},\qquad \beta+\frac{\delta_M}{\varepsilon}+b_M(L)+e_0\le1,$$

then $H|_{\mathcal{S}}\ge-\varepsilon I$. One may replace $b_M(L)$ by the explicit $d_M/a(L)$.

Proof. The residual lemma gives $G\le\overline{G}$. Hence $U=R_\varepsilon Q_0\overline{G}^{-1/2}$ is a contraction. The finite operator is at most

$$U[\overline{G}^{1/2}\overline{C}\,\overline{G}^{1/2}]U^*\le\beta UU^*\le\beta I.$$

The last step uses $\beta\ge0$. Apply (10) and the receiving equivalence. □

A full residual in this statement is a vector in the complete receiving space. Its projection onto a finite trial space is insufficient. Likewise the covariance E^*E must be enclosed as a positive matrix; entrywise optimistic endpoints are not Loewner bounds. The same condition applies to numerical errors in C and G. Evaluating $b_M(L)$ also requires coverage of the entire high half-line, rather than sampling its multiplier.

5 Sharpness of the raw frequency tail

The improved arithmetic error does not improve the raw-output bound itself.

Theorem 5.1 (Sharp raw tail)**.** *For the realization of Section 2 and every fixed $\varepsilon > 0$,*

$$\lim_{L\to\infty} \sqrt{a(L)}\, \|(I - Q_L)M_s(A+\varepsilon)^{-1/2}\|_{\mathcal{K}\to L^2} = 1. \tag{13}$$

The same limit holds with A replaced by its reference on $\mathcal{S} = Z^{\perp} \cap \mathcal{K}$ under the following additional hypotheses: Z is a closed subspace of $\ker H$, its orthogonal projection Π preserves the form domain, $Z \subset \mathrm{Dom}(A)$, and $A\Pi$ is a bounded compact operator on $\mathcal{K}$. The reference on $\mathcal{S}$ is the operator of the restricted form. This second assertion is conditional on these precise null-space facts, rather than a claim that a selected family exhausts the kernel.

Proof. The energy estimate (6) gives the upper bound $a(L)^{-1/2}$ in both cases. Choose nonzero $\phi \in C_c^{\infty}(\mathbb{R})$. Fix two compact smooth functions whose matrix of the two constraint rows is invertible, as in the density argument. Correct $e^{iNu}\phi(u)$ by their linear combination to obtain $v_N \in \mathcal{C}$. Each correction coefficient decreases faster than every inverse power of N: integrate the compact smooth row integrands repeatedly by parts. In particular v_N is bounded in L^2 and converges weakly to zero.

Set $b = \|s\phi\| > 0$. The Fourier transform of $s\phi$ is Schwartz. By (3), translation of that transform gives

$$\mathfrak{a}[v_N] = b^2 \log N + O(1), \qquad \|(I - Q_{N/2})sv_N\| = b + o(1). \tag{14}$$

Here is the needed control of the energy asymptotic. On the translated region $|\xi - N| \le N/2$, $a(\xi) = \log N + O(1)$. On its complement, use $a(\xi) \le C + \log(1 + |\xi|)$ and the Schwartz tail. The lost mass times $\log N$ and the remaining logarithmic integral tend to zero faster than a fixed inverse power of N. This gives the upper and lower leading coefficient b^2. Fixed smooth correction terms have rapidly decreasing form norm, and their mixed terms are bounded by form Cauchy–Schwarz. The norm assertion follows from the translated Schwartz mass outside $[-N/2, N/2]$.

For the full constrained space take $w_N = v_N$. Under the additional null-space hypotheses take $w_N = (I - \Pi)v_N$. Compactness gives $\|A\Pi v_N\| \to 0$. Consequently $\mathfrak{a}[\Pi v_N] \to 0$ and

$$\mathfrak{a}[w_N] = \mathfrak{a}[v_N] + o(1),$$

since the mixed term equals $\langle v_N, A\Pi v_N\rangle$. Furthermore

$$\|(I - Q_{N/2})s\Pi v_N\| \le \sqrt{\mathfrak{a}[\Pi v_N]/a(N/2)} = o((\log N)^{-1/2}).$$

Thus (14) holds for w_N as well, and its norm remains bounded. Form-domain preservation makes this a valid restricted-form vector. In either case the unit vector

$$u_N = \frac{(A_{\mathcal{S}} + \varepsilon)^{1/2} w_N}{\sqrt{\mathfrak{a}[w_N] + \varepsilon\|w_N\|^2}}$$

has image under J equal to sw_N divided by the same denominator. Take $N = 2L$. Equations (14) give a tail norm at least $(1 + o(1))/\sqrt{\log L}$. Since $a(L)/\log L \to 1$, this matches the upper bound and proves the theorem. □

Remark 5.2 (A sufficient compact-null mechanism). The compactness hypothesis in the conditional assertion has a source-specific sufficient condition in the present model. The map $M_s : \mathcal{V} \to L^2$ is compact: bounded form sets have complete spatial tails bounded by σ_R and frequency tails bounded by $a(L)^{-1/2}$, and a finite space–frequency rectangle is Hilbert–Schmidt. Finite weighted-shift truncations then show $B(A+I)^{-1/2}$ compact, using (5). If a closed null subspace has $Z \subset \mathrm{Dom}(A)$ and $A\Pi = -B\Pi$, its null-energy identity bounds $(A+I)^{1/2}\Pi$. Therefore

$$A\Pi = -B(A+I)^{-1/2}(A+I)^{1/2}\Pi$$

is compact. The null identities and form-domain preservation are hypotheses to verify before compression.

6 Localization budgets and the remaining sign problem

For fixed M and ε, the coarse frequency term in (9) is at most a tolerance η once $a(L) \geq d_M/\eta$. Since $a(L) = \log L + O(1)$, this supplies $\log L = O(d_M/\eta)$ as $\eta \downarrow 0$. In contrast, approximating J first in norm places $a(L)^{-1/2}$ inside the first-order product error and gives a sufficient cutoff budget of inverse-square tolerance order. Theorem 5.1 shows that this raw-output loss cannot be removed by a better uniform raw-frequency estimate. The improvement in Theorem 3.1 instead uses exact cross-frequency cancellation.

This compares sufficient cutoff bounds. A fixed M leaves a nonzero arithmetic error; achieving an arbitrarily small complete budget requires increasing M. Parameters can be chosen in the order M, L, R, d, paying respectively the arithmetic tail, high band, spatial tail and finite-phase remainder. The logarithmic frequency reference can still make the resulting rank enormous.

A sign advance requires a certified signed finite matrix and its full residual covariance satisfying that inequality, and a threshold conclusion requires the corresponding limiting argument as $\varepsilon \to 0$. Neither a small arithmetic tail, a finite compression, this localization improvement nor a known-null projection alone proves that condition.

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